Int. J. Heat Mass Transfer. Vol. 17. pp. 529–540. Pergamon Press 1974. Printed in Great Britain

# HEAT TRANSFER AND FRICTION COEFFICIENTS IN SMOOTH AND ROUGH TUBES WITH DILUTE POLYMER SOLUTIONS

PAUL M. DEBRULE* and ROLF H. SABERSKY
Division of Engineering and Applied Science, California Institute of Technology, Pasadena, California, U.S.A.

(*Received* 23 *June* 1973)

**Abstract**—A series of experiments were performed for the purpose of determining the heat transfer, and friction coefficients in smooth and rough tubes for dilute solutions of Polyethelyne oxide ("Polyox") in water. Tests were conducted with solutions containing 10 ppm and 50 ppm of the polymer respectively. Typical results for the smooth tube showed that the friction coefficient was reduced by a factor of 3 and the heat transfer coefficient by a factor of 5. For the rough tube the coefficients were reduced even more drastically. The effectiveness of the polymer decreased in the range of high Reynolds numbers and this decrease was ascribed to the deterioration of the polymer.

## NOMENCLATURE

| | |
|---|---|
| $C$, | polymer concentration; |
| $c$, | specific heat; |
| $C_F$, | friction coefficient; |
| $C_H$, | heat transfer coefficient; |
| $D$, | tube diameter; |
| $j$, | Colburn factor; |
| $k$, | thermal conductivity; |
| $n$, | constant in Deissler's equation; |
| $Pr$, | Prandtl number, $c\mu/k$; |
| $q$, | heat transfer rate per unit area; |
| $Re$, | Reynolds number, $DV/\nu$; |
| $T$, | temperature; |
| $u$, | velocity; |
| $u^*$, | dimensionless velocity, $u/\sqrt{\tau_0/\rho}$; |
| $V$, | average pipe velocity; |
| $y$, | coordinate normal to wall; |
| $\varepsilon$, | roughness dimension; |
| $\varepsilon^*$, | dimensionless roughness $(\varepsilon\sqrt{\tau_0/\rho})/\nu$; |
| $\varepsilon_M$, | turbulent momentum exchange coefficient; |
| $\varepsilon_H$, | turbulent heat exchange coefficient; |
| $\nu$, | kinematic viscosity; |
| $\rho$, | density; |
| $\tau_0$, | wall shear stress. |

## 1. INTRODUCTION

BY FAR the largest part of experimental work in heat transfer has been performed with rather simple, single-component fluids and most often the fluid has been air or water. This, of course, is entirely logical because heat transfer to water and air is of most immediate interest in every day life as well as in industrial processes. However, as our technology becomes more and more sophisticated, design information on less common and more complex fluids is needed. An example of this type of development is the fact that a considerable effort has already been devoted to the heat transfer to liquid metals. Water solutions of polymers have received widespread study in recent years, after it was discovered (Toms [1]) that tremendous reductions of turbulent pipe friction could be achieved by using dilute solutions of various polymeric additives. The practical possibilities of using this phenomenon—the Toms phenomenon—in several applications certainly justify the numerous efforts to achieve a better understanding of the mechanisms involved in drag reduction. Much of the early work was carried out at naval establishments with a view toward application in this field. Perhaps the most immediate one is the drag reduction of ships, which could be brought about by injecting small amounts of the solute into the boundary layer around the surfaces of the vessel. Other applications include the use of polymer solutions instead of pure water for fire fighting. In addition thought is being given to the use of polymer additives in order to reduce the friction in pumping oils, slurries, and irrigation water.

Most of the experiments conducted so far have been concerned with the determination of friction. Heat transfer data are still rather scarce. Most probably, such information will, however, be needed in the future. In some applications, for instance, water to which a polymer has been added for the purpose of friction reduction, may also be involved in a heat exchange process; in other instances the polymer (or a similar

---

*Presently at Tetra Tech, Inc., Pasadena, California, U.S.A.

drag reducer) may be dissolved in the water unintentionally as a contaminant; and as a further example, certain fluids involved in food processing, in the production of paper, and the manufacture of chemicals may inherently have drag reducing characteristics. Furthermore, it is of importance to study the effect of roughened surfaces on heat and momentum transfer, as roughness often occurs naturally either from the manufacturing process or from subsequent wear.

The types of problems mentioned above have been the stimulus for the present research program. The program was designed so that it would yield information on friction and heat transfer coefficients which might be directly useful in certain engineering applications. In addition it was hoped that the interpretation of the data would also allow an insight into the mechanism by which the polymer affects the friction and heat transfer. The experiments were limited to hydrodynamically and thermally fully established pipe flow with negligible radial fluid property variations and to roughness elements of a fixed type. In this way, it should be possible to express the results in terms of relationships between the friction coefficient $C_F$ and the heat transfer coefficient $C_H$ on the one hand, and the Reynolds number, $Re$, Prandtl number, $Pr$ roughness ratio, $\varepsilon/D$, the concentration, $C$, and a parameter characterizing the shape of the polymer on the other. The experiments were designed to allow a systematic variation over a fairly wide range of $Re$, $Pr$ and $\varepsilon/D$.

## 2. PREVIOUS WORK

As mentioned before, a great number of investigations have been conducted into the friction-reducing characteristics of polymers and other additives. It has been firmly established that, depending on the flow conditions, the surface stress may be reduced by factors of three or more. The drag reducing effect occurs in turbulent flow only, and the phenomenon may be explained by the concept that the additives increase the effective thickness of the viscous layer adjacent to the wall. The results of experiments in which turbulent fluctuations were measured directly, tended to support this concept.

A very careful and extensive review of the literature on the fluid mechanics of polymer solutions has been prepared by Hoyt [2]. For further information the reader is referred to this excellent paper which also contains a very complete bibliography.

As to heat transfer with drag reducing fluids the literature is not nearly as extensive and the range of variables (Reynolds number, Prandtl number, different drag reducing agents, their concentration range and wall roughness) which has been investigated is still limited. One of the first heat transfer studies in this field was that by Gupta, Metzner and Hartnett [3] and their work clearly shows the reduction in heat transfer coefficient which accompanies the reduction in friction. In these experiments the additive was a polyacrylamide (ET 597, Dow Chemical Co.) which was used at concentrations from 0·01 to 0·8 per cent in a 1·82 cm i.d. smooth tube. The results show that at a given flow rate, the reduction in heat transfer rates is greater than the reduction in pressure drop. Also presented in the paper is a method of analysis of the data (following Reichardt's approach) which allows some insight into the heat transfer mechanism, particularly close to the wall where the most important changes in velocity and temperature occur. The same polymer at concentrations of 600 and 1000 ppm was used by Marrucci and Astarita [4] and their data can be reduced to

$$j = C_H \times Pr^{2/3} = 0{\cdot}6\, C_F/2$$

where $j$ is the Colburn factor.

McNally [5] working with Polyox WSR 301 at concentrations of 2, 10, 20 and 40 ppm in a 1·9 cm i.d. smooth pipe over a 25000–167000 Reynolds number range concluded that the heat transfer coefficients were reduced as drastically as the friction factor, the Colburn analogy $j = C_H \times Pr^{2/3} = C_F/2$ satisfactorily relating the heat and momentum transport in these solutions, provided a normalization is made with respect to the Reynolds number at the onset of drag reduction. Smith, K. A. Keuroghlian, Virk and Merrill [6], using Polyox WSR 301 as well (10 ppm) found that only the heat transfer coefficient corresponding to the maximum heat transfer obtained with the polymer solutions obeyed a Colburn type analogy

$$C_H \times Pr^{0{\cdot}6} = C_F/2.$$

Experimental data of $C_F$ and $C_H$ with Polyox WSR 301 for concentrations of 5, 12·5 and 100 ppm at temperatures of 17·8 and 4·5°C are also available in [7, 8]. This set of data, however, contains considerable scatter. In addition, the range of Reynolds number is limited to about 30000.

A set of heat transfer data for a rough pipe was obtained by White [9] who used a threaded pipe of extreme roughness. The results show that Polyox reduces the friction factor considerably, but leads to an even greater reduction in the Stanton number. Aside from the experimental work a certain number of analyses have been conducted in which friction coefficients as well as heat transfer coefficients are predicted on the basis of various models of the boundary layer.

## 3. TEST INSTALLATION

A simplified diagram of the test facility as well as a more detailed sketch of the test section are shown in Figs. 1 and 2. The flow through the test section is

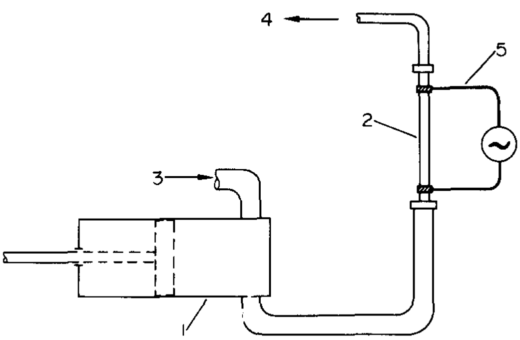

FIG. 1. Schematic diagram of test installation. The numbered items represent: 1. Hydraulic cylinder; 2. Tube being tested; 3. Fill line; 4. Discharge, line to reservoir; and 5. Electric power connection.

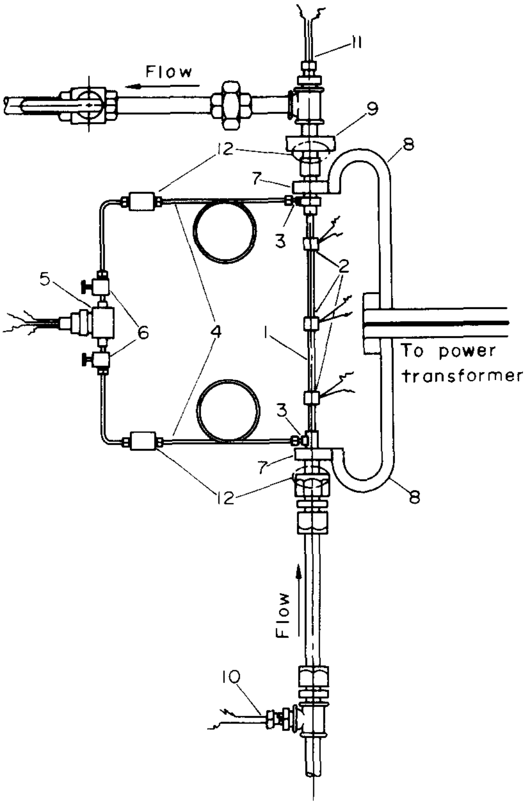

FIG. 2. Detail of test section. The numbered items represent: 1. Tube being tested; 2. Wall thermocouples; 3. Pressure taps; 4. Pressure lines; 5. Pressure transducers; 6. Valves; 7. Copper electrodes; 8. Copper bus bars; 9. Mixing chamber; 10. Inlet immersion thermocouple; 11. Outlet immersion thermocouple; and 12. Electric insulation sections.

supplied from a hydraulic cylinder operated by a variable speed motor through a linear actuator. This simple displacement method of supplying the flow was preferred to a pump in order to reduce possible mechanical degradation of the solutions. At the same time, the displacement of the piston in the cylinder is used to determine the flow rate. After passing through the test section, the fluid is discharged into a reservoir tank. The apparatus is designed so that it can operate at pressures exceeding one atmosphere. The solutions were prepared either in the reservoir tank or in auxiliary drums directly connected to the cylinder by a flexible hose.

The test section is heated by passing alternating current through the wall of the tube. As the electrical resistance of the wall is only about 0·002Ω, a relatively large current (of the order of 1000 A) is required, which is supplied through a special set of transformers. The heat transfer rate to the test fluid is determined from the electrical dissipation in the pipe wall.

The pipes for the present experiments are the same as those used by Dipprey in his work with distilled water [10]. All of the pipes are made of nickel, a material of thermal conductivity sufficiently well established to permit accurate calculation of inner wall temperature from measurements on the outer wall; their inside diameter is about 0·95 cm. The rough tubes were constructed by electroplating nickel onto a sand-covered mandrel and by subsequently dissolving the mandrel with chemicals leaving a pure nickel shell which served as the test tube. More details on the basic tube construction are presented in [10].

The roughness of the tube is characterized by their hydraulic behavior in the "fully rough" regime and the values are noted in Table 1. The friction drop is determined from pressure readings taken at smooth portions of the wall provided just upstream and downstream of the heated length. Temperatures on the outer surface of the wall are measured by thermocouples and the corresponding temperatures at the inside wall are computed taking into account the conduction and heat dissipation in the wall. These data together with the flow and heat transfer rate provide the principle information for determining the friction and heat transfer coefficient.

Table 1. Tube dimensions

| | Designation | Length | Wall thickness | Roughness* ratio |
|---|---|---|---|---|
| "Smooth" tube | E-3 | 0·44 m | 0·051 cm | — |
| "Rough" tube | A-4 | 0·44 m | 0·047 cm | 0·049 |

*Equivalent sand grain roughness based on friction coefficient in the fully rough regime.

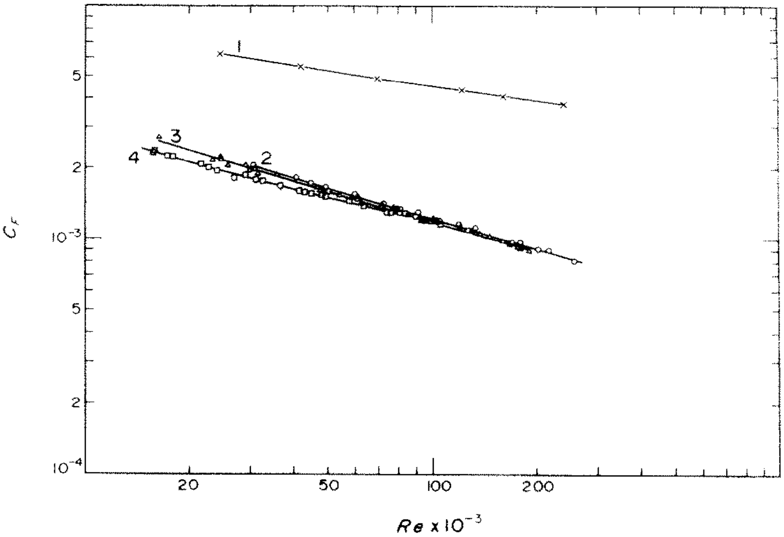

FIG. 3. Friction coefficient vs Reynolds number. 50 ppm Polyox solution. Smooth tube. Curve 1 is for pure water at all $Pr$ (for comparison). Curves 2, 3 and 4 correspond to $Pr$ of 4·38, 6·16 and 10·3.

## 4. EXPERIMENTAL RESULTS

### (a) *The smooth tube*

The experimental results obtained for the smooth tube are summarized in Figs. 3–6, which shows the friction and heat transfer coefficients respectively, as functions of the Reynolds number, for two different concentrations.

Let us first consider the friction coefficient for a 50 ppm solution as given in Fig. 3. Curve No. 1 on this graph corresponds to pure water and is in excellent agreement with the data in the literature, particularly with those by Dipprey [10]. This pure water curve has been given for reference purposes only. The data for the 50 ppm solution are presented by three curves which, however, are separated only slightly. The data for each curve were taken at a particular Prandtl number, $Pr$, which was obtained by operating at a particular temperature. The temperature for $Pr = 4{\cdot}38$ was $T = 40{\cdot}5\,^{\circ}\mathrm{C}$, for example and that for $Pr = 10{\cdot}3$ was $T = 4{\cdot}5\,^{\circ}\mathrm{C}$. Although the $Pr$ does not have any

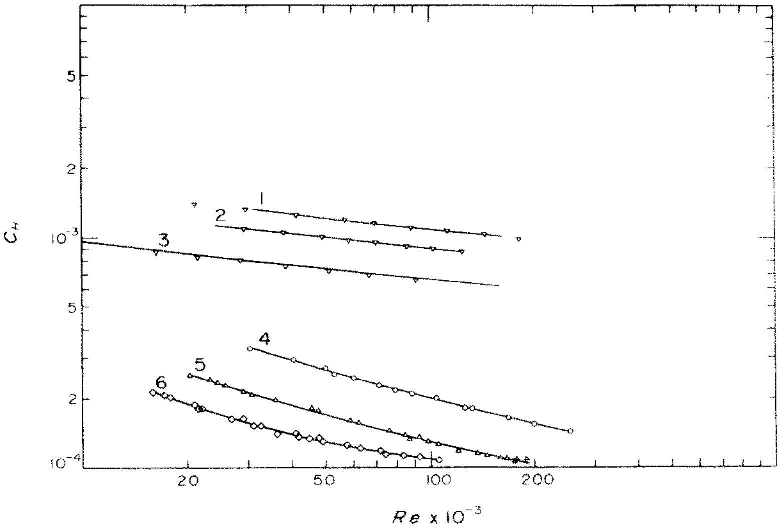

FIG. 4. Heat transfer coefficient vs Reynolds number. 50 ppm Polyox solution. Smooth tube. Curves 1, 2 and 3 are for pure water (for comparison) and correspond to $Pr$ of 4·38, 6·16 and 10·3. Curves 4, 5 and 6 are for the Polyox solution at $Pr$ 4·38, 6·16 and 10·3.

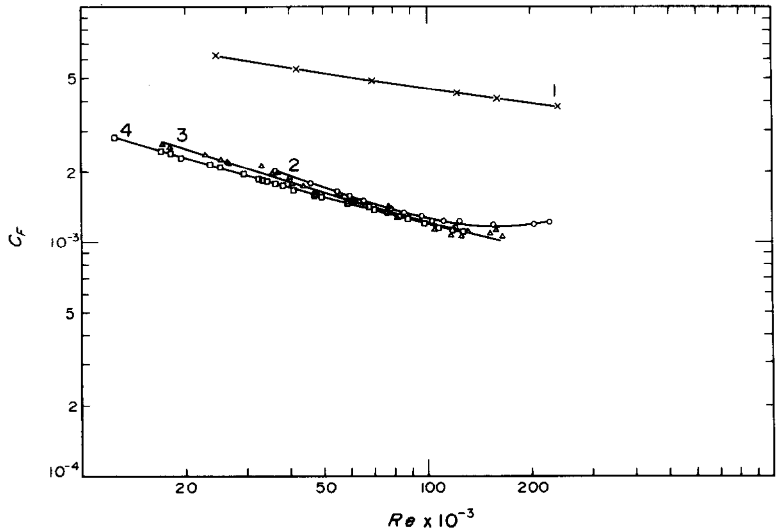

FIG. 5. Friction coefficient vs Reynolds number. 10 ppm solution. Smooth tube. Curve 1 is for pure water at all *Pr* (for comparison). Curves 2, 3 and 4 correspond to *Pr* of 4·38, 6·16 and 10·3.

direct relation to the friction coefficient, the temperature does seem to have a slight influence on the effectiveness of the solution. The reduction in friction is slightly less at the higher temperature but for the present tests the differences are very small. A significant reduction in friction is indicated by all three curves. Selecting a *Re* of 80 000 as typical the ratio of the friction coefficient of the solution to that of clear water is about 0·28. This finding is in general agreement with those of earlier experiments (1).

The friction coefficient for the 10 ppm solution (Fig. 5), shows results very similar to those for the more concentrated one. The only noticeable difference is a change of slope in the curves at $Re > 150000$ particularly for the data taken at the highest temperature ($T = 40{\cdot}5°\text{C}$, $Pr = 4{\cdot}38$). This change is attributed to a deterioration of the polymers. This deterioration is believed to be related to the wall shear, which increases with the *Re*, and it is plausible that the solution of low concentration at the highest temperature should be most susceptible.

The principal purpose of the present work was, of

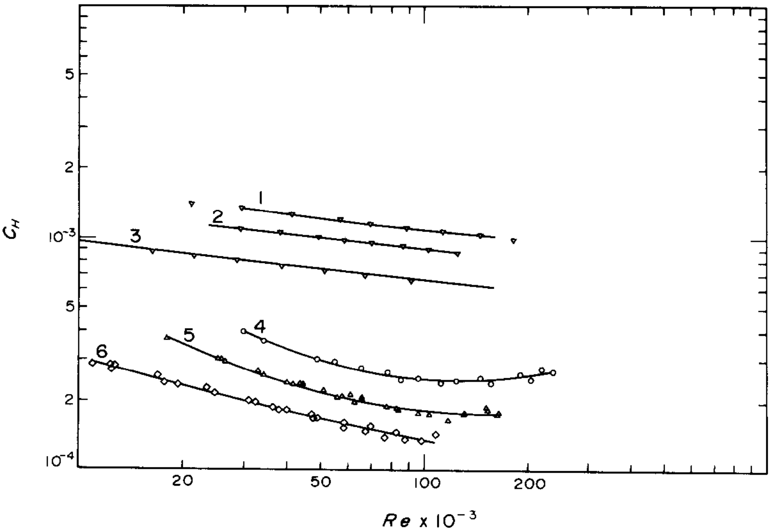

FIG. 6. Heat transfer coefficient vs Reynolds number. 10 ppm Polyox solution. Smooth tube. Curves 1, 2 and 3 are for pure water (for comparison) and correspond to *Pr* of 4·38, 6·16 and 10·3. Curves 4, 5 and 6 are for the Polyox solution at *Pr* 4·38, 6·16 and 10·3.

course, the investigation of the heat transfer characteristics and those for the smooth tube at a concentration of 50 ppm are summarized in Fig. 4. In this graph the heat transfer coefficient $C_H$ is given as a function of *Re* for three different *Pr* numbers. For comparison the data obtained for pure water are also shown. It is seen that the addition of Polyox greatly reduces the heat transfer coefficient. As an example one may again consider the values at $Re = 80\,000$. At this *Re* the heat transfer coefficient for all three *Pr* is reduced to about 20 per cent of its value for clear water. Of interest also is the ratio $2C_H/C_F$ which gives some measure of the relative performance for heat transfer as compared to friction. For a $Pr = 4{\cdot}38$ at $Re = 80\,000$ this ratio changes from 0·45 for clear water to 0·37 for the 50 ppm solution, indicating that the heat transfer mechanism is even more strongly affected by the polymer than the friction mechanism. The corresponding figures for $Pr = 6{\cdot}16$ are 0·38 and 0·28, and for $Pr = 10{\cdot}3$ they are 0·28 and 0·21.

The heat transfer coefficient for the 10 ppm solution is given in Fig. 6. The reduction in this coefficient is almost as great as for the more concentrated solution, except at the higher *Re*, where the effects which were ascribed to deterioration begin to be noticeable.

### (b) *The rough tube*

Although tests were conducted with two tubes of different roughness, it will be sufficient for our purposes to present the data for the roughest tube only. Again the more concentrated solution will be considered first. The friction coefficient is given in Fig. 7 and the heat transfer coefficient in Fig. 8. The values for clear water are shown for comparison. From Fig. 7 it is then seen that the friction is reduced drastically, at least for *Re* below about 80 000. In that region the reduction amounts to a factor of about 6 which exceeds the reduction ratio obtained in the smooth tube. At *Re* above 80 000 the coefficient gradually increases. This trend again is thought to be connected with deterioration of the polymer and will be discussed further in Section 5. The reduction ratio of the heat transfer coefficient at the lower range of *Re* ($Re < 80\,000$) is unexpectedly large, and is of the order of 10 for all three Prandtl numbers. The cause for the increase of $C_H$ at higher Reynolds numbers is believed to be the same as that responsible for the increase in $C_F$, and both begin at about the same value of *Re*. The behavior of $C_F$ and $C_H$ for the 10 ppm solution is very similar to that for the more concentrated solution. At the lower Reynolds numbers the friction coefficient $C_F$, is reduced by a factor of about 6 and the heat transfer coefficient $C_H$ by a factor of nearly 10. The trend of these coefficients to increase begins already at a *Re* of 50 000 compared with $Re = 80\,000$ for the 50 ppm solution. As before, it is suspected that the low concentration is more susceptible to deterioration.

## 5. DISCUSSION OF RESULTS

### (a) *The smooth tube*

The fact that small amounts of Polyox dissolved in water drastically reduce friction and heat transfer has been shown by many experimenters. The present results confirm these findings and at the same time provide

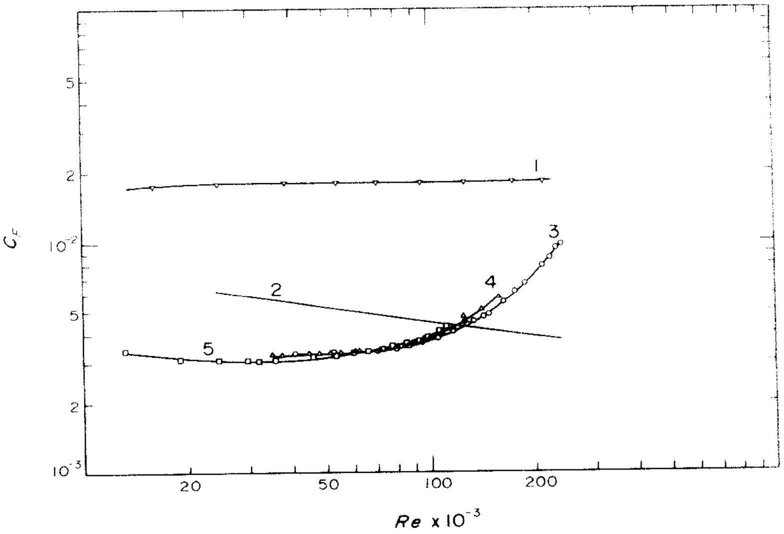

FIG. 7. Friction coefficient vs Reynolds number. 50 ppm Polyox solution. Rough tube. Curve 1 is for pure water at all *Pr* (for comparison). Curve 2 (also for comparison) is for pure water and the smooth tube. Curves 3, 4 and 5 are for the Polyox solution at *Pr* of 4·38, 6·16 and 10·3.

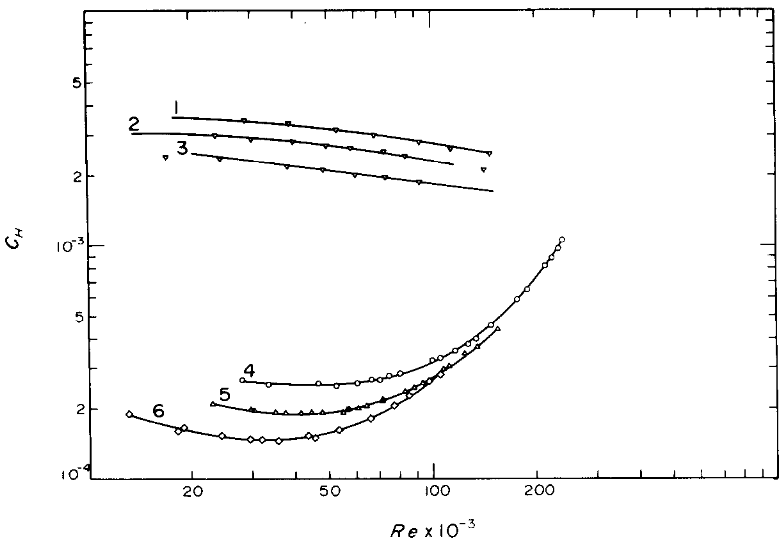

FIG. 8. Heat transfer coefficient vs Reynolds number. 50 ppm Polyox solution. Rough tube. Curves 1, 2 and 3 are for pure water (for comparison) and correspond to $Pr$ of 4·38, 6·16 and 10·3. Curves 4, 5 and 6 are for the Polyox solution at $Pr$ 4·38, 6·16 and 10·3.

a rather complete set of data covering two concentrations, three Prandtl numbers and a range of Reynolds numbers. The results, however, may also be used to gain some information on the mechanism by which the polymers act. Generally, the effect of the polymers has been explained by postulating that the long chain molecules reduce the turbulence near the wall and increase the thickness of the effective viscous layer. To examine this point of view one may analyze the relation

$$\frac{\dfrac{C_F}{2C_H}\dfrac{\varepsilon_H}{\varepsilon_M}-1}{\left(\dfrac{\varepsilon_H}{\varepsilon_M}Pr-1\right)\sqrt{C_F/2}} = \int_0^{u_c^*}\frac{\mathrm{d}u^*}{1+\dfrac{\varepsilon_H}{\varepsilon_M}Pr\dfrac{\varepsilon_M}{\nu}}. \tag{1}$$

In this equation $\varepsilon_H$ and $\varepsilon_M$ represent the turbulent exchange coefficients for heat and momentum respectively and $u_c^*$ is the velocity at the center of the pipe in dimensionless terms. This expression may be derived directly from equations for the shear and heat transfer near the wall which are

$$\frac{\tau_0}{\rho} = (\nu+\varepsilon_M)\frac{\mathrm{d}u}{\mathrm{d}y} \tag{2}$$

and

$$\frac{q}{\rho c} = \left(\frac{k}{\rho c}+\varepsilon_H\right)\frac{\mathrm{d}T}{\mathrm{d}y} \tag{3}$$

with the usual definitions of the friction coefficient, $C_F$, and the heat transfer coefficient, $C_H$. The assumptions involved are minor and not central to the present discussion.

Consider now as a typical set of data *for pure water* those obtained at $Re = 100\,000$ and $Pr = 6{\cdot}16$. Assuming $\varepsilon_H/\varepsilon_M$ to be unity and using the experimental data for $C_F$ and $C_H$, the left hand side of equation (1) becomes equal to 7·9. The right hand side is of course expected to equal this number by virtue of the equality. (The expression increases to 8·7 if $\varepsilon_H/\varepsilon_M$ is taken to be 1·5 and decreases to 3·1 for $\varepsilon_H/\varepsilon_M = 0{\cdot}5$.) Taking next the experimental values of $C_F$ and $C_H$ for a *50 ppm polymer solution*, the left hand side of equation (1) increases to 28·1, again with $\varepsilon_H/\varepsilon_M$ set equal to 1·0. (Setting $\varepsilon_H/\varepsilon_M$ equal to 1·5 and 0·5 changes the expression only to 29·1 and 23·8 respectively.) It is not reasonable to postulate much wider changes in the ratio $\varepsilon_H/\varepsilon_M$. The value of the integral on the right hand side will, of course, have to match that of the left hand side. Examining this integral it is seen that no reasonable variation in $\varepsilon_H/\varepsilon_M$ can account for the increase that the value of this integral has to sustain when the polymer solution takes the place of pure water. Studying the other features of the integral, it has to be realized first that the velocity increases with the distance from the wall, and as $u^*$ increases the term $\varepsilon_M/\nu$ appearing in the denominator of the integral in equation (1) becomes very large; the integrand then approaches zero rapidly, and beyond a certain distance from the wall, the value of the integral remains essentially constant. This general behavior applied both to clear water and to the polymer solution. One may conclude then, that a change in the value of the integral must be caused by changes in the variation of $\varepsilon_M/\nu$ in the wall region. More specifically, starting at the wall. $\varepsilon_M$ must increase much more slowly in the polymer solution than in the pure water: in other words, in the polymer solution the viscous stresses will remain of importance in a layer near the wall which is much thicker than the corresponding layer for pure water. Thus the viscous region in the polymer solution exceeds that for pure water. This reasoning, even though indirect, gives a rather strong substantiation to the prevailing assumption that the polymers act to reduce the turbulent fluctuations near the wall and to increase the thickness of the viscous layer.

After this generalized analysis of the results let us next examine a more detailed flow model in the light of the experimental findings. The model to be considered is of the simple "two layer" type. In the layer near the wall the velocity is assumed to be linearly related to the distance from the wall. Beyond that point the velocity is given by the expression

$$u^* = A \ln y^* + B + \Delta u^*. \qquad (4)$$

In this expression $A$ and $B$ are constants, $A$ representing the slope of the familiar logarithmic profile. In general one would have to expect $A$ to depend on some characteristic of the polymer and on the concentration (3), however, in view of the available measurements it seems justifiable to assume that $A$ does not change from its value for pure water, and a constant value of 2·46 was, therefore, selected for $A$ in our considerations. The constant $B$ was set equal to 6·01, which corresponds to the constant in the logarithmic profile of pure water. Finally, $\Delta u^*$ in equation (4) is a positive value representing the shift in the velocity profile caused by the presence of the polymer. With these assumptions and explanations, the logarithmic profile is parallel to the one for pure water but shifted towards higher values of the velocity by a constant increment. The velocity very near the wall is still assumed to be a linear function of distance and as $\Delta u^*$ increases the intersection of linear profile with the logarithmic one will take place at larger values of the distance $y^*$. This intersection may be taken as an indication of the extent of the viscous layer and the viscous layer, therefore, increases as $\Delta u^*$ increases. For pure water $\Delta u^*$ is equal to zero, and the expression reduces to the familiar logarithmic profile.

$$u^* = A \ln y^* + B. \qquad (5)$$

A relation between the friction coefficient and the Reynolds number may be derived from this latter expression by integrating over the pipe radius. Neglecting the deviation from the logarithmic profile in the intermediate vicinity of the wall one obtains the well known results

$$\sqrt{\frac{2}{C_F}} = A \ln Re \sqrt{C_F/8} - \beta + B \qquad (6)$$

where $\beta$ is the dimensionless difference between the centerline velocity and the average velocity, which difference is independent of $B$. For polymer solutions an analogous approach may be followed, which, making use of equation (4) leads to

$$\sqrt{\frac{2}{C_F}} = A \ln Re \sqrt{\frac{C_F}{8}} - \beta + B + \Delta u^*. \qquad (7)$$

For the present development this expression together with the experimental results for $C_F$ were used to determine $\Delta u^*$. (One may recall that in the derivations for $C_F$ the logarithmic profile is generally assumed to extend to the wall. With the thickened viscous layers which occur in polymer solutions the validity of this assumption may be questioned. For this reason an alternate relation was developed for equation (7), taking into account the linear portion of the velocity profile near the wall. A comparison showed that no corrections to equation (7) were required in our case.)

Having determined $\Delta u^*$, the heat transfer process will now be examined. Equation (1) will again be used as a basis, with $\varepsilon_H/\varepsilon_M$ set equal to unity.

In principal the right hand side of equation (1) can be evaluated if the distribution of $\varepsilon_M$ is known, and $\varepsilon_M$ in turn is obtainable from the relation

$$\frac{\tau_0}{\rho} = (\nu + \varepsilon_M)\frac{\mathrm{d}u}{\mathrm{d}y} \qquad (2)$$

provided $\mathrm{d}u/\mathrm{d}y$ is known with sufficient accuracy. The assumption of a linear velocity profile throughout the viscous layer leads to the conclusion that $\varepsilon_M = 0$ in that region. Although such an assumption was permissible when discussing friction it removes from consideration the key factor characterizing the differences between transport mechanisms of the fluids under consideration. To obtain a better representation of the flow in the region near the wall, Deissler many years ago proposed the following empirical relationship for $\varepsilon_M$ in the viscous layer:

$$\frac{\varepsilon_M}{\nu} = n^2 \frac{uy}{\nu}\left[1 - \exp\left(-n^2 \frac{uy}{\nu}\right)\right]. \qquad (8)$$

Setting $n = 0{\cdot}124$ he was able to predict very accurately the heat transfer coefficients (as well as mass transfer coefficients) for a wide range of $Pr$ (or Schmidt numbers). Because of this success Deissler's approach was followed for the interpretation of the present data; instead of taking his value of $n$, however, $n$ was allowed to assume different values depending on the concentration, the temperature (related to $Pr$ in the present experiments) and the Reynolds number. For each flow condition (as determined by $Re$, $Pr$, and concentration), the right hand side of equation (1) was then evaluated using equation (8) as an expression for $\varepsilon_M$. Various values of $n$ were tried and the particular value of $n$ was then determined which would fit best the left hand side of the equation which was evaluated from the experimental results.

The results of the computation show that the value of $n$ is indeed drastically reduced by the polymer indicating a reduction in the turbulent transfer coefficients $\varepsilon_M$ or $\varepsilon_H$. Instead of $n = 0{\cdot}124$, which was the value for pure water, typical values of $n$ for the polymer solutions are between 0·030 and 0·040 for the 50 ppm solution and between 0·040 and 0·050 for the 10 ppm solution. This drastic change in $n$ then adds further to the credibility of the concept that the polymers act to reduce turbulent fluctuations near the wall.

Thus it has been possible to obtain some insight into the detailed mechanism by which the polymer acts, from measurements of only macroscopic quantities such as heat transfer rates, pressure drops, and flow rates.

### (b) *The rough tubes*

The friction coefficient of the rough tube for a 50 ppm solution is shown in Fig. 7 and the curve obtained with pure water in the same tube is also shown. For pure water $C_F$ is essentially independent of *Re* for the entire range indicating that the flow is in the "fully rough" regime. It has been demonstrated that for Newtonian fluids at least, fully rough flow takes place if the roughness height exceeds the thickness of the viscous sublayer by a factor of about 6, where the viscous sublayer is defined as the intersection of the linear velocity profile with the logarithmic one as it exists in a smooth tube. Taking into account the increase in this layer brought about by the polymer, fully rough flow should be expected under the present conditions for $Re > 100\,000$. Any transition to the fully rough behavior should take place at *Re* below this value and the friction coefficient should be constant for $Re > 100\,000$. Instead of reaching a constant level beyond this *Re*, however, the coefficient of friction begins to increase steadily, and the polymer seems to lose its effectiveness in reducing drag. Because of the high *Re* at which this increase begins, it is doubted that the observed behavior represents a transition to fully rough flow and the explanation for the increase in $C_F$ is most likely a deterioration of the polymer. This explanation is the more plausible, as the shear velocities at which the loss in effectiveness occurs is greater than about 1·0 m/s, and it has been reported that Polyox solutions are degraded significantly for shear velocities over 0·5 m/s.

The data for the 10 ppm solution are shown in Fig. 9. The *Re* beyond which fully rough flow might be expected is $Re > 80\,000$. Again the rise in friction coefficient starts beyond that value and again deterioration of the Polymer is a probable explanation. For the highest values of *Re* in the present series, the Polymer seems to have entirely lost its drag reducing ability.

Of more basic interest, possibly, are the results for the range below about $Re = 80\,000$. As discussed

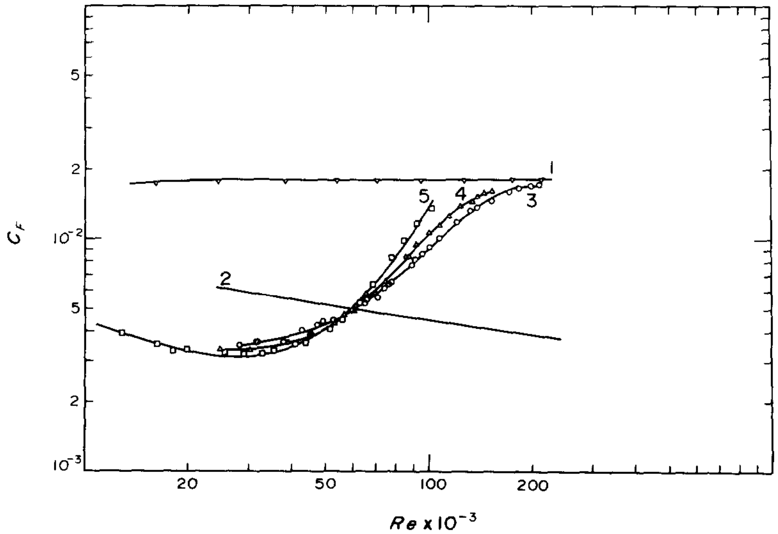

FIG. 9. Friction coefficient vs Reynolds number. 10 ppm Polyox solution. Rough tube. Curve 1 is for pure water at all *Pr* (for comparison). Curve 2 (also for comparison) is for pure water and the smooth tube. Curves 3, 4 and 5 are for the Polyox solution at *Pr* of 4·38, 6·16 and 10·3.

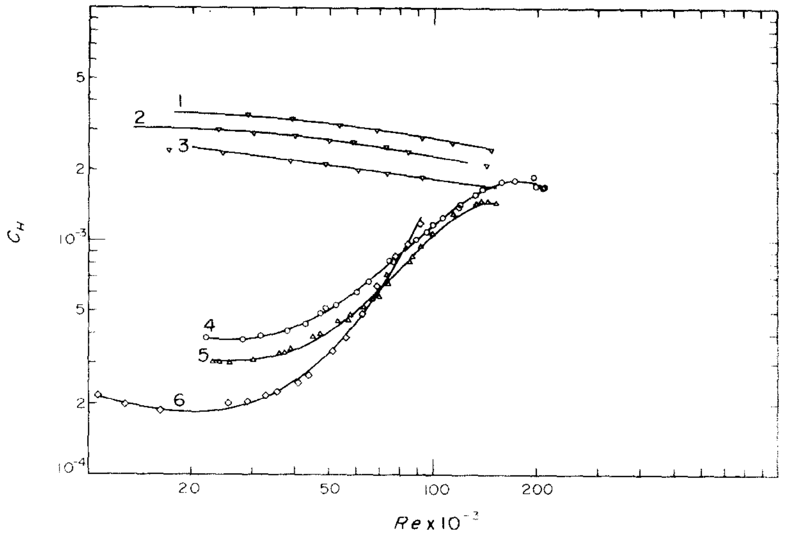

FIG. 10. Heat transfer coefficient vs Reynolds number. 10 ppm Polyox solution. Rough tube. Curves 1, 2 and 3 are for pure water (for comparison) and correspond to $Pr$ of 4·38, 6·16 and 10·3. Curves 4, 5 and 6 are for the Polyox solution at $Pr$ of 4·38, 6·16 and 10·3.

above, the flow type in this range is at least close to the "fully rough" condition, and nevertheless a very substantial drag reduction is achieved. Now, the general concept of fully rough flow is that the viscous layer is completely destroyed and that the turbulent flow characteristic of the core extends to the tips of the roughnesses. The wall shear, of course, has to be sustained by the rough surface and it is largely made up of drag over the surface elements. The fact that drag reduction is obtained for rough surfaces with polymer solutions, indicates that the flow between and around the roughnesses is affected by the presence of the polymer. For pure water the pipe friction coefficient will depend only on $\varepsilon/D$ and become independent of $\varepsilon^*$, where

$$\varepsilon^* = \varepsilon \sqrt{\frac{\tau_0}{\rho}} \Big/ \nu$$

for sufficiently large values of this parameter, which is formed from a roughness height $\varepsilon$, the shear velocity $\sqrt{\tau_0/\rho}$, and the kinematic viscosity, $\nu$. Whether or not the behavior for a polymer solution is similar is not definitely known at this time. It is likely, however, that the friction coefficient for the solution is not only dependent on $\varepsilon^*$ and $\varepsilon/D$, but also on a parameter containing a length of the polymer. If the friction coefficient for the polymer solution were to reach an asymptotic value, this value still could differ from that for the pure fluid.

The behavior of the heat transfer coefficient at the two concentrations is generally similar. The increase of $C_H$ at the higher $Re$ may again be ascribed to deterioration. In the lower portion of the experimental range perhaps the most remarkable finding is that the roughness does not increase the heat transfer coefficient beyond that for a smooth tube and in fact for Reynolds numbers smaller than about 50 000 it is below the value for the smooth tube. This behavior contrasts with that of the friction coefficient, as $C_F$ for the rough tube consistently exceeds that for the smooth tube. It also contrasts with the experience obtained with pure water for which roughness increases both $C_F$ and $C_H$ even though not to the same extent. One may infer from this fact that the flow around the roughnesses must be particularly unfavorable to heat transfer. A series of experiments (11) in which the flow over a set of cavities was visualized for values of $\varepsilon^*$ that are typical for pipe flows, illustrates the type of phenomenon that might occur. In these experiments the cavities were to simulate the roughness elements and it was seen that for values of $\varepsilon^*$ in neighborhood of the "fully rough" conditions, periodic large-scale fluctuations promoted the exchange of fluid between the cavities and the main flow. Such an exchange would certainly promote heat transfer, and the presence of polymers might inhibit exchanges of this kind.

## SUMMARY AND CONCLUSIONS

A fairly extensive set of data was obtained for the heat transfer and friction coefficients of Polyox solutions in a smooth tube. A rather wide range of Reynolds numbers was covered and experiments were conducted at three different Prandtl numbers. All experiments were performed in the same installation using the same measurement techniques which facilitates the assessment of the trends caused by the several parameters. An analysis of the data was carried out with the purpose of obtaining an indication of the mechanism by which the polymer influences the flow. The results tend to substantiate the thought that the action of the polymers consist of damping the turbulent motion near the wall and of increasing the effective viscous sublayer.

The results with the rough tube were of particular interest. Both the friction coefficient as well as the heat transfer coefficient were reduced drastically, that is by factors of 6 and 10 respectively. Furthermore, this reduction seemed to persist even in the range in which the flow was thought to be hydraulically "fully rough". This indicates that the polymer also influences the flow in between and around the roughness elements. The results at high *Re* are somewhat clouded because the polymer is believed to deteriorate at the high shear rates which correspond to these flow conditions.

*Acknowledgement*—The work reported herein has been supported in part by the Department of the Navy, Contract # N 66001-72-C-0009 and by the E. I. DuPont de Nemours and Company, Inc. This support is gratefully acknowledged. The authors also wish to express their sincere thanks to Dr. J. Hoyt, Dr. D. F. Dipprey and Dr. A. J. Acosta for their interest in the project and their help in planning and directing the research.

## REFERENCES

1. B. A. Toms, Some observations on the flow of linear polymer solutions through straight tubes at large Reynolds numbers, *Proc. 1st Int. Congr. on Rheology*, Vol. II, pp. 135–141. North Holland (1949).
2. J. W. Hoyt, The effect of additives on fluid friction. *Trans. Am. Soc. Mech. Engrs* **94D**(2) (1972).
3. M. K. Gupta, A. C. Metzner and J. P. Hartnett, Turbulent heat transfer characteristics of viscoelastic fluids, *Int. J. Heat Mass Transfer* **10**, 1211–1224 (1967).
4. G. Marrucci and G. Astarita, Turbulent heat transfer in viscoelastic liquids, *IEC Fundamentals* **6**(3), 470–471 (1967).
5. W. A. McNally, Heat and momentum transport in dilute polyethylene oxide solutions, Naval Underwater Weapons Res. & Engineering Station, Newport, Rhode Island, TR No. 44 (Dec. 1968).
6. K. A. Smith, G. H. Keuroghlian, P. S. Virk and E. W. Merrill, Heat transfer to drag-reducing polymer solutions, *A.I.Ch.E. Jl* **15**(2), 294–297 (1969).
7. R. G. Howard, Heat and momentum transfer in drag-reducing solutions, Naval Ship Res. & Dev. Lab., Annapolis, Report 3226 (Oct. 1970).
8. R. G. Howard and D. M. McCrory, The correlation between heat and momentum transfer for solutions of drag-reducing agents, Naval Ship Res. & Dev. Lab., Annapolis, Report 3232 (Jan. 1971).
9. A. White, Heat transfer characteristics of dilute polymer solutions in fully rough pipe flow. *Nature, Lond.* **227**, 486–487 (1970).
10. D. F. Dipprey and R. H. Sabersky, Heat and momentum transfer in smooth and rough tubes at various Prandtl numbers, *Int. J. Heat Mass Transfer* **6**, 329 (1963).
11. H. W. Townes and R. H. Sabersky, Experiments of the flow over a rough surface, *Int. J. Heat Mass Transfer* **9**, 729–738 (1966).

## COEFFICIENTS DE TRANSFERT DE CHALEUR ET DE FROTTEMENT DANS DES TUBES LISSES ET RUGUEUX AVEC DES SOLUTIONS DILUEES DE POLYMERE

**Résumé**—Une série d'expériences à été pratiquée sur des solutions aqueuses diluées d'oxyde de polyéthylène ("Polyox"), dans le but de déterminer les coefficients de transfert thermique et de frottement dans des tubes lisses et rugueux. Les essais ont été faits avec des solutions contenant respectivement 10 et 50 ppm de polymère. Des résultats typiques pour le tube lisse ont montré que le coefficient de frottement est réduit par un facteur de 3 et que le coefficient de transfert thermique l'est par un facteur de 5. Quant au tube rugueux, les coefficients sont réduits beaucoup plus radicalement. L'efficacité du polymère diminue dans le domaine des grands nombres de Reynolds et cette diminution est attribuée à la détérioration du polymère.

## WÄRMEÜBERGANGS- UND REIBUNGSKOEFFIZIENTEN FÜR VERDÜNNTE POLYMER-LÖSUNGEN IN GLATTEN UND RAUHEN ROHREN

**Zusammenfassung**—Zur Bestimmung der Wärmeübergangs- und Reibungskoeffizienten für verdünnte Lösungen von Polyäthylenoxid ("Polyox") in Wasser, in glatten und rauhen Rohren wurde eine Reihe von Experimenten durchgeführt. Die Versuche wurden mit Lösungen von 10 und 50 ppm Polymerbestandteilen ausgeführt. Typische Ergebnisse für das glatte Rohr zeigten, daß der Reibungskoeffizient um einen Faktor 3 und der Wärmeübergangskoeffizient um einen Faktor 5 vermindert wurde. Für rauhe Rohre wurden die Koeffizienten sogar noch drastischer vermindert. Die Wirkung des Polymers ließ im Bereich hoher Reynolds-Zahlen nach und die Verminderung wurde dem Zerfall des Polymers zugeschrieben.

# КОЭФФИЦИЕНТЫ ТЕПЛООБМЕНА И ТРЕНИЯ В ГЛАДКИХ И ШЕРОХОВАТЫХ ТРУБАХ ПРИ ТЕЧЕНИИ РАЗБАВЛЕННЫХ РАСТВОРОВ ПОЛИМЕРОВ

**Аннотация** — В опытах определены коэффициенты теплообмена и трения в гладких и шероховатых трубах при течении разбавленных водных растворов полиэтиленоксида ("Polyox") с концентрациями 10 и 50 ppm. Для гладких труб оказалось, что коэффициент трения уменьшается в три раза, а коэффициент теплообмена — в пять раз. В случае шероховатых труб эти коэффициенты снизились еще сильнее. Эффективность полимеров уменьшалась при больших числах Рейнольдса, что относится за счёт деградации полимера.